



Racionális kitevőjű hatványok

1. feladat

Egyszerűbb alakra hozzuk a következő kifejezéseket. A végeredményben minden betű egyszer szerepel, és nem tartalmaz zárójelet.

a)

$$\begin{aligned} \frac{a^{\frac{2}{3}} \cdot \sqrt[4]{a^3}}{\sqrt[3]{a^2} \cdot \sqrt{a^{-5}}} &= \\ &= \frac{a^{\frac{2}{3}} \cdot a^{\frac{3}{4}}}{a^{\frac{2}{3}} \cdot a^{\frac{-5}{2}}} = \frac{a^{\frac{3}{4}}}{a^{\frac{-5}{2}}} = \frac{a^{\frac{3}{4}}}{a^{\frac{-5}{2}}} = \\ &= a^{\frac{3}{4} - (-\frac{5}{2})} = a^{\frac{3}{4} + \frac{5}{2}} = a^{\frac{3}{4} + \frac{10}{4}} = a^{\frac{13}{4}} = \sqrt[4]{a^{13}} \end{aligned}$$



b)

$$\begin{aligned}
 & \frac{(\sqrt[3]{a^{-2}} \cdot \sqrt{a^3})^{-\frac{2}{7}}}{(a^{\frac{1}{7}} \cdot \sqrt[4]{a^{-1}})^{-\frac{3}{4}}} = \\
 & = \frac{(a^{-\frac{2}{3}} \cdot a^{\frac{3}{2}})^{-\frac{2}{7}}}{(a^{\frac{1}{7}} \cdot a^{-\frac{1}{4}})^{-\frac{3}{4}}} = \\
 & = \frac{(a^{-\frac{2}{3} + \frac{3}{2}})^{-\frac{2}{7}}}{(a^{\frac{1}{7} + (-\frac{1}{4})})^{-\frac{3}{4}}} = \frac{(a^{-\frac{4}{6} + \frac{9}{6}})^{-\frac{2}{7}}}{(a^{\frac{4}{28} - \frac{7}{28}})^{-\frac{3}{4}}} = \frac{(a^{\frac{-4+9}{6}})^{-\frac{2}{7}}}{(a^{\frac{4-7}{28}})^{-\frac{3}{4}}} = \frac{(a^{\frac{5}{6}})^{-\frac{2}{7}}}{(a^{\frac{-3}{28}})^{-\frac{3}{4}}} = \\
 & = \frac{a^{\frac{5}{6} \cdot (-\frac{2}{7})}}{a^{-\frac{3}{28} \cdot (-\frac{3}{4})}} = \frac{a^{-\frac{10}{42}}}{a^{\frac{9}{112}}} = \frac{a^{-\frac{5}{21}}}{a^{\frac{9}{112}}} = \\
 & = a^{-\frac{5}{21} - \frac{9}{112}} = a^{-\frac{80}{336} - \frac{27}{336}} = a^{-\frac{80-27}{336}} = a^{-\frac{107}{336}} = a^{-\frac{107}{336}} = \\
 & = \sqrt[336]{a^{-107}} = \frac{1}{\sqrt[336]{a^{107}}}
 \end{aligned}$$



c)

$$\begin{aligned}
 & \left(27 \cdot a^{-\frac{1}{2}} \cdot \sqrt[3]{b \cdot a^{\frac{3}{2}} \cdot \sqrt[4]{b^{\frac{4}{3}}}} \right)^{\frac{1}{3}} = \\
 & = 27^{\frac{1}{3}} \cdot \left(a^{-\frac{1}{2}} \right)^{\frac{1}{3}} \cdot \left(\sqrt[3]{b \cdot a^{\frac{3}{2}} \cdot \sqrt[4]{b^{\frac{4}{3}}}} \right)^{\frac{1}{3}} = \\
 & = \sqrt[3]{27} \cdot a^{-\frac{11}{2 \cdot 3}} \cdot \left(\left(b \cdot a^{\frac{3}{2}} \cdot b^{\frac{4 \cdot 1}{3 \cdot 4}} \right)^{\frac{1}{3}} \right)^{\frac{1}{3}} = \\
 & = \sqrt[3]{27} \cdot a^{-\frac{1}{6}} \cdot \left(\left(b \cdot a^{\frac{3}{2}} \cdot b^{\frac{1}{3}} \right)^{\frac{1}{3}} \right)^{\frac{1}{3}} = \\
 & = 3 \cdot a^{-\frac{1}{6}} \cdot \left(b^{\frac{1}{3}} \cdot a^{\frac{3 \cdot 1}{2 \cdot 3}} \cdot b^{\frac{1 \cdot 1}{3 \cdot 3}} \right)^{\frac{1}{3}} = \\
 & = 3 \cdot a^{-\frac{1}{6}} \cdot \left(b^{\frac{1}{3}} \cdot a^{\frac{1}{2}} \cdot b^{\frac{1}{9}} \right)^{\frac{1}{3}} = \\
 & = 3 \cdot a^{-\frac{1}{6}} \cdot b^{\frac{1 \cdot 1}{3 \cdot 3}} \cdot a^{\frac{1 \cdot 1}{2 \cdot 3}} \cdot b^{\frac{1 \cdot 1}{9 \cdot 3}} = \\
 & = 3 \cdot a^{-\frac{1}{6}} \cdot b^{\frac{1}{9}} \cdot a^{\frac{1}{6}} \cdot b^{\frac{1}{27}} = \\
 & = 3 \cdot a^{-\frac{1}{6} + \frac{1}{6}} \cdot b^{\frac{1}{9} + \frac{1}{27}} = \\
 & = 3 \cdot a^0 \cdot b^{\frac{3}{27} + \frac{1}{27}} = 3 \cdot 1 \cdot b^{\frac{4}{27}} = 3 \cdot b^{\frac{4}{27}} = \\
 & = 3 \cdot \sqrt[27]{b^4}
 \end{aligned}$$